General Perfect Aggregation of Industries in Input-Output Models

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Abstract

The traditional conditions for aggregation of input-output models are shown to be too narrow, since they pose unnecessary conditions on the intra-branch flows; the same objection applies to the standard measures of aggregation bias. The conditions of perfect aggregation are therefore generalized, and results concerning price models are added; this leads to revised aggregation bias measures enabling new possibilities of "good" aggregations.

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Contents

1. Introduction

The conditions for an input-output model with aggregated industries to be consistent with a more fundamental input-output model containing a larger number of industries have been the subject of many contributions since the initiating note by Hatanaka(1952). Hatanakas conditions for such a "perfect aggregation" were soon modified by McManus(1956)[2], who stressed the need to distinguish between gross production and net production and to account for this difference when comparing the solutions of the aggregated and original models; he derived the necessary and sufficient conditions in each case. Ever since, the standard approach has been to consider perfect aggregation of gross productions, even though McManus clearly states that the conditions for perfect aggregation of net productions are weaker.[3] In this paper I will show that such a "net production approach" is the proper one to use and that, consequently, the true conditions for perfect aggregation are considerably weaker than those derived from the standard approach. The same objection applies to the standard measures of "aggregation bias" used for empirical work, since such measures are based on a too narrow criterion for unbiased aggregation.

2. Prerequisites

The basis of the models is the input-output flow tables in value units in a base year; such tables are outlined in table 1.

Table 1. Notation for the fundamental and aggregated i-o tables.

Fundamental				Aggregated			
	Industries	Final demands	Sum		Industries	Final Demands	Sum
Primary inputs	$\mathbf{y}'$	$\mathbf{0}$	$\mathbf{y}'\mathbf{i}$	Primary inputs	$\mathbf{y}^{*\prime}$	$\mathbf{0}$	$\mathbf{y}^{*\prime}\mathbf{i}$
Industries	$\mathbf{A}$	$\mathbf{e}$	$\mathbf{x}$	Industries	$\mathbf{A}^*$	$\mathbf{e}^*$	$\mathbf{x}^*$
Sum	$\mathbf{x}'$	$\mathbf{i}'\mathbf{e}$		Sum	$\mathbf{x}^{*\prime}$	$\mathbf{i}'\mathbf{e}^*$	

Note: The symbol **i** is used for the summation vector. The variables **y** and **x** are assumed positive, while **e** is assumed nonnegative (so that some elements can be 0).

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[2]The term "perfect aggregation" was introduced by Theil(1957). Other terms used in the literature for the same concept are "acceptable", "consistent", "admissible", "intrinsic" and "unbiased" aggregation.

[3]A fairly recent bibliography is given in Olsen(1993).

The top bar symbol ($\bar{\ }$) is consistently used to denote a matrix of coefficients, i. e. a matrix with elements defined as shares of totals of the base year input-output table. The hat symbol (^) on a vector indicates a reshaping of the vector into a square diagonal matrix. The subscript $_0$ is consistently used to denote matrices or vectors in base year value units. Thus, $\mathbf{\bar{A}}=\mathbf{A_0}(\mathbf{\hat{x}_0})^{-1}$ and (as a notational shortcut) $\mathbf{\bar{y}}=\mathbf{\hat{y}_0}\mathbf{\hat{x}_0}^{-1}$.

The fundamental quantity model with n industries is given by

$$\mathbf{x} = \mathbf{\bar{A}x} + \mathbf{e} = (\mathbf{I}-\mathbf{\bar{A}})^{-1}\mathbf{e} \tag{1}$$

$$\mathbf{y} = \mathbf{\bar{y}x} = \mathbf{\bar{y}}(\mathbf{I}-\mathbf{\bar{A}})^{-1}\mathbf{e} \tag{2}$$

provided that $(\mathbf{I}-\mathbf{\bar{A}})$ is regular. Likewise, the aggregated quantity model with n^* industry groups (main branches) is given by

$$\mathbf{x}^* = (\mathbf{I}-\mathbf{\bar{A}}^*)^{-1}\mathbf{e}^* \tag{3}$$

$$\mathbf{y}^* = \mathbf{\bar{y}}^*\mathbf{x}^* = \mathbf{\bar{y}}^*(\mathbf{I}-\mathbf{\bar{A}}^*)^{-1}\mathbf{e}^* \tag{4}$$

where, using the ($n^*\times n$) aggregation matrix $\mathbf{G}$ such that $\mathbf{x}_0^*=\mathbf{Gx_0}$ and $\mathbf{A}_0^*=\mathbf{GA_0G'}$,

$$\mathbf{\bar{A}}^* = \mathbf{A}_0^*\mathbf{\hat{x}}_0^{*-1} = \mathbf{G\bar{A}\bar{W}'_x} \quad \text{where} \tag{5}$$

$$\mathbf{\bar{W}_x} = \mathbf{\hat{x}}_0^{*-1}\mathbf{G\hat{x}_0} \tag{6}$$

$$\mathbf{\bar{y}}^* = \mathbf{G\hat{y}_0G'\hat{x}}_0^{*-1} = \mathbf{G\bar{y}\bar{W}'_x} \tag{7}$$

$$\mathbf{e}^* = \mathbf{Ge} \tag{8}$$

The aggregator $\mathbf{G}$ is (as usual) taken to be a simple grouping matrix, which means that g_{ij}=1 if industry j is in branch i, g_{ij}=0 otherwise; flows are therefore aggregated simply by summation, while indexes such as columns of input coefficients (and prices below) are aggregated by weighting using matrices such as $\mathbf{\bar{W}_x}$ (in which $w_{ij}=x_j/x_i^*$ if industry j is in branch i, $w_{ij}=0$ otherwise).[4] The standard problem of aggregation in input-output models is caused by the fact that in general the gross production $\mathbf{x}^*$ from (3) is not equal to $\mathbf{Gx}$ from (1), even though this equality holds by definition in the base year.

[4]Simple and weighted grouping matrices such as $\mathbf{G}$ and $\mathbf{\bar{W}_x}$, respectively, are treated in more detail in Olsen(1993). Note that $\mathbf{G\bar{W}'_x}=\mathbf{I}$ and that, therefore, $\mathbf{\bar{W}'_x}$ is a generalized inverse of $\mathbf{G}$. A third type of grouping matrix, such as $\mathbf{\bar{W}_a}$ introduced in (13.b) fulfilling $\mathbf{\bar{W}_a\bar{W}'_x}=\mathbf{I}$, will be termed double-weighted grouping matrices.

The early contributions adressed the question whether to use gross production $\mathbf{x}$ or to use only net production defined as production minus the supplies from the industry to itself, i.e. (in the case of the aggregated system)[5]

$$\mathbf{x}_n^* = \mathbf{x}^* – \mathbf{a}^* = (\mathbf{I}–\bar{\mathbf{a}}^*)\mathbf{x}^* \tag{9}$$

where $\mathbf{a}^*$ and $\bar{\mathbf{a}}^*$ are a vector and a matrix of the diagonal elements in matrices $\mathbf{A}^*$ and $\bar{\mathbf{A}}^*$, respectively. It is clear from (9) that in the model (1) net production is merely a rescaling of gross production. Since, in a net production system, the intra-branch supplies are eliminated, the system can be formed from table 1 by setting the diagonal elements of matrix $\mathbf{A}^*$ to 0 and simultaneously subtracting them from the marginals; the coefficient matrix of the net system is found like in (5), which amounts to a rescaling of the columns of the non-diagonal part of $\bar{\mathbf{A}}^*$, using (9):

$$\tilde{\mathbf{A}}^* = (\mathbf{A}_0^* – \hat{\mathbf{a}}_0^*)\hat{\mathbf{x}}_{n0}^{*\,–1} = (\bar{\mathbf{A}}^* – \bar{\mathbf{a}}^*)(\mathbf{I}–\bar{\mathbf{a}}^*)^{–1} \tag{10}$$

The choice of formulation - gross or net - of the system is of no real significance, since the two systems produce the same relations between primary inputs and final demands:

$$\bar{\mathbf{y}}^*(\mathbf{I}–\bar{\mathbf{A}}^*)^{–1} = \hat{\mathbf{y}}_0^*(\hat{\mathbf{x}}_0^* – \mathbf{A}_0^*)^{–1} = \tilde{\mathbf{y}}^*(\mathbf{I}–\tilde{\mathbf{A}}^*)^{–1} \tag{11}$$

using (4) and the notation $\tilde{\mathbf{y}}^* = \hat{\mathbf{y}}_0^* \hat{\mathbf{x}}_{n0}^{*\,–1}$. In other words, the choice of formulation is merely a question of a choice between the identical matrices $\hat{\mathbf{x}}_0^* – \mathbf{A}_0^*$ and $\hat{\mathbf{x}}_0^* – \hat{\mathbf{a}}_0^* – (\mathbf{A}_0^* – \hat{\mathbf{a}}_0^*)$; but of course, the vector of production used as an intermediate variable is scaled differently in the two cases, as in (9).

The question of gross or net production is nevertheless important in relation to the aggregation problem. This is because a number of flows, which are between different industries in the fundamental system, become purely internal flows in the main branches of the aggregated system. Following McManus(1956) I will address this question by decomposing the coefficient matrix $\bar{\mathbf{A}}$ into two parts: The part $\bar{\mathbf{A}}_d$ containing flows that will end up in the diagonal of the aggregated matrix, and its complementary part $\bar{\mathbf{A}}_c = \bar{\mathbf{A}} – \bar{\mathbf{A}}_d$, so that from (5)

$$\bar{\mathbf{A}}^* = \mathbf{G}(\bar{\mathbf{A}}_c + \bar{\mathbf{A}}_d)\bar{\mathbf{W}}'_x = \mathbf{G}\bar{\mathbf{A}}_c\bar{\mathbf{W}}'_x + \mathbf{G}\bar{\mathbf{A}}_d\bar{\mathbf{W}}'_x = \bar{\mathbf{A}}_c^* + \bar{\mathbf{a}}^* \tag{12}$$

where $\bar{\mathbf{a}}^* = \mathbf{G}\bar{\mathbf{A}}_d\bar{\mathbf{W}}'_x$ is the diagonal part of $\bar{\mathbf{A}}^*$ and $\bar{\mathbf{A}}_c^* = \mathbf{G}\bar{\mathbf{A}}_c\bar{\mathbf{W}}'_x$ is the off-diagonal part. Note that matrix $\bar{\mathbf{A}}_d$ will be block-diagonal, whereas matrix $\bar{\mathbf{A}}_c$ will contain zeros

[5]Similar formulae hold for the fundamental system, but they are of no relevance here. The fundamental system can be interpreted as a net or gross system according to taste; all formulae are valid in either case.

in the same diagonal blocks. McManus shows that perfect aggregation in the "net production sense" that $\mathbf{x}_n^*=\mathbf{G}(\mathbf{I}-\bar{\mathbf{A}}_d)\mathbf{x}$ requires conditions on matrix $\bar{\mathbf{A}}_c$ only, while perfect aggregation in the "gross production sense" that $\mathbf{x}^*=\mathbf{Gx}$ requires additional conditions on matrix $\bar{\mathbf{A}}_d$. His findings will be in the core of the present paper, since I will claim that the "net production" approach is the relevant one and that the correct condition for perfect aggregation therefore is

$$\mathbf{x}_n^*=\mathbf{G}\mathbf{x}_n \qquad \text{where } \mathbf{x}_n = (\mathbf{I}-\bar{\mathbf{A}}_d)\mathbf{x} \qquad \Leftrightarrow \tag{13.a}$$

$$\mathbf{x}^*=\bar{\mathbf{W}}_a\mathbf{x} \qquad \text{where } \bar{\mathbf{W}}_a = (\mathbf{I}-\bar{\mathbf{a}}^*)^{-1}\mathbf{G}(\mathbf{I}-\bar{\mathbf{A}}_d) \tag{13.b}$$

using (9), rather than the traditional condition $\mathbf{x}^*=\mathbf{Gx}$.[6] In other words, gross productions should be aggregated using a weighting rather than simple summation, thereby adjusting for possibly different intensities of intra-branch flows in those fundamental industries grouped into the same main branch; the element *(k,j)* of matrix $\bar{\mathbf{W}}_a$ is $\sum_{i\in k}(1-\bar{a}_{ij})/(1-\bar{a}_k^*)$ for $j\in$ group k, 0 otherwise.

It will prove appropriate to define the "net" version of the weight matrix $\bar{\mathbf{W}}_x$ from (6) as[7]

$$\tilde{\mathbf{W}}_x = \hat{\mathbf{x}}_{n0}^{*\,-1}\mathbf{G}\hat{\mathbf{x}}_{n0} = (\mathbf{I}-\bar{\mathbf{a}}^*)^{-1}\bar{\mathbf{W}}_x(\mathbf{I}-\bar{\mathbf{A}}_d') \tag{14}$$

3. Perfect aggregation of industries in the quantity model

The idea behind the concept of perfect aggregation is that the solutions for the endogenous variables from the aggregated model should be the same as the aggregated solutions for the endogenous variables from the fundamental model. But since the input-output model determines both primary inputs, gross production and net production, we are left with three criteria which are possibly inconsistent. I will show that the "net production" approach of McManus is the only criterion yielding the same conditions for perfect aggregation for all three types of endogenous variables.

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[6]The matrix $\bar{\mathbf{W}}_a$ is a common (double-weighted) grouping matrix though it does not immediately appear so. This is because $\mathbf{G}\bar{\mathbf{A}}_d=\mathbf{G}\hat{\mathbf{w}}_d$ where $\mathbf{w}_d'=\mathbf{i}'\bar{\mathbf{A}}_d$ is a vector showing for each industry the coefficient for total inputs from industries belonging to the same main branch, so that element *j* of $\mathbf{w}_d'$ is $\sum_{i\in k}\bar{a}_{ij}$ for $j\in$ group *k*. Thus, $\bar{\mathbf{W}}_a=(\mathbf{I}-\bar{\mathbf{a}}^*)^{-1}\mathbf{G}(\mathbf{I}-\hat{\mathbf{w}}_d)$. Note that $\bar{\mathbf{W}}_a\bar{\mathbf{W}}_x'=\mathbf{I}$.

[7]The matrix $\tilde{\mathbf{W}}_x$ is a common weighted grouping matrix though it does not immediately appear so. This is because $\bar{\mathbf{A}}_d\bar{\mathbf{W}}_x' = \hat{\mathbf{w}}_n\mathbf{G}'$ where $\mathbf{w}_n = \bar{\mathbf{A}}_d\bar{\mathbf{W}}_x'\mathbf{i}$ is a vector showing, for each industry, the coefficient for inputs produced in that industry per unit production in the main branch to which it belong, so that element *i* of $\mathbf{w}_n$ is $\sum_{j\in k} a_{ij}/x_{0k}^*$ for $i\in$ group *k*. Thus, $\mathbf{x}_{n0} = (\mathbf{I}-\hat{\mathbf{w}}_n)\mathbf{x}_0$ and $\tilde{\mathbf{W}}_x = (\mathbf{I}-\bar{\mathbf{a}}^*)^{-1}\bar{\mathbf{W}}_x(\mathbf{I}-\hat{\mathbf{w}}_n)$.

Definition 1. The aggregation of the quantity model (1)-(2) into (3)-(4) is (weakly) *perfect* for a given set of vectors of final demand $\mathbf{e}$ if and only if

$$\mathbf{y}^* = \mathbf{G}\mathbf{y} \qquad (15)$$

i.e. that the two models yield the same vector of primary inputs by the aggregated industries for that set.

This concept of perfect aggregation is the proper operationalization of the basic idea outlined above. However, it is possible to define a stronger concept requiring that that endogenous variables from the *fundamental* model can be determined from the aggregated model:

Definition 2. The aggregation of the quantity model (1)-(2) into (3)-(4) is *superperfect* for a given set of vectors of final demand if and only if

$$\mathbf{y} = \overline{\mathbf{W}}_y'\mathbf{y}^* \quad \text{, where } \overline{\mathbf{W}}_y = \hat{\mathbf{y}}_0^{*-1}\mathbf{G}\hat{\mathbf{y}}_0 \qquad (16)$$

i.e. that the primary inputs are proportional in the groups of $\mathbf{G}$ and, therefore, that the two models yield the same vector of primary inputs by the fundamental industries for that set. It is immediately clear, that

- superperfect aggregation is a special case of perfect aggregation, since (16) implies (15) but not vice versa
- the aggregation (5)-(8) is superperfect by definition for all vectors $\mathbf{e}$ proportional to $\mathbf{e_0}$
- no nontrivial aggregation can be superperfect for arbitrary $\mathbf{e}$, due to the smaller dimensionality of the aggregated variables.

Following the basic idea I will, in general, seek the conditions for (weakly) perfect aggregation for arbitrary (nonnegative) aggregated vectors $\mathbf{e}^*=\mathbf{G}\mathbf{e}$, but results for other domains and for superperfect aggregation are developed in the process.

Lemma 1. An aggregation of the quantity model is superperfect if and only if

$$\mathbf{x}=\overline{\mathbf{W}}_x'\mathbf{x}^*$$

i.e. the productions are proportional in the groups of $\mathbf{G}$.

Proof: $\mathbf{y} = \overline{\mathbf{W}}_y'\mathbf{y}^* = \hat{\mathbf{y}}_0\mathbf{G}'\hat{\mathbf{y}}_0^{*-1}\hat{\mathbf{y}}_0^*\hat{\mathbf{x}}_0^{*-1}\mathbf{x}^* = \overline{\mathbf{y}}\overline{\mathbf{W}}_x'\mathbf{x}^* \quad \Leftrightarrow \quad \mathbf{x} = \overline{\mathbf{W}}_x'\mathbf{x}^*$

using (2), (16), (4), (6), the definitions $\overline{\mathbf{y}}^*=\hat{\mathbf{y}}_0^*\hat{\mathbf{x}}_0^{*-1}$, $\overline{\mathbf{y}}=\hat{\mathbf{y}}_0\hat{\mathbf{x}}_0^{-1}$ and the fact that $\overline{\mathbf{y}}$ is invertible.

□

Note that (16) implies $\mathbf{x}^*=\mathbf{Gx}=\bar{\mathbf{W}}_\mathbf{a}\mathbf{x}$, since $\mathbf{G}\bar{\mathbf{W}}'_\mathbf{x}=\bar{\mathbf{W}}_\mathbf{a}\bar{\mathbf{W}}'_\mathbf{x}=\mathbf{I}$, see (13.b) and note 5.

Theorem 1. Superperfect aggregation of quantities. An aggregation of the quantity model (1),(2) into (3),(4) is superperfect for arbitrary $\mathbf{e}^*$ if and only if there exists a matrix $\mathbf{Z}$ such that the conditions (17) and (18) are both satisfied:

$$\mathbf{e} = \mathbf{Z}\mathbf{e}^* \qquad \text{arbitrary } \mathbf{e}^* \tag{17}$$

$$\mathbf{Z}(\mathbf{I}-\bar{\mathbf{A}}^*) = (\mathbf{I}-\bar{\mathbf{A}})\bar{\mathbf{W}}'_\mathbf{x} \tag{18}$$

Proof: From lemma 1, an aggregation is superperfect if and only if

$$\mathbf{x} = \bar{\mathbf{W}}'_\mathbf{x}\mathbf{x}^* \qquad \Leftrightarrow$$

$$(\mathbf{I}-\bar{\mathbf{A}})^{-1}\mathbf{e} = \bar{\mathbf{W}}'_\mathbf{x}(\mathbf{I}-\bar{\mathbf{A}}^*)^{-1}\mathbf{G}\mathbf{e}$$

Since the matrix on the left side has rank n while the matrix on the right side has rank $n^*<n$, this equation can only hold if $\mathbf{e}$ obeys at least $(n-n^*)$ linear restrictions. Since we assume that $\mathbf{e}^*$ can be arbitrary, there must exist a matrix $\mathbf{Z}$ such that $\mathbf{e} = \mathbf{Z}\mathbf{e}^*$ for arbitrary $\mathbf{e}^*$. Substituting for $\mathbf{e}$ above and using (8) we obtain

$$(\mathbf{I}-\bar{\mathbf{A}})^{-1}\mathbf{Z} = \bar{\mathbf{W}}'_\mathbf{x}(\mathbf{I}-\bar{\mathbf{A}}^*)^{-1} \qquad \Leftrightarrow$$

$$\mathbf{Z}(\mathbf{I}-\bar{\mathbf{A}}^*) = (\mathbf{I}-\bar{\mathbf{A}})\bar{\mathbf{W}}'_\mathbf{x}$$

□

If all the elements in $\hat{\mathbf{e}}_\mathbf{0}^*$ are nonzero, then a valid choice for $\mathbf{Z}$ is $\bar{\mathbf{W}}'_\mathbf{e}$ where[8]

$$\bar{\mathbf{W}}_\mathbf{e} = \hat{\mathbf{e}}_\mathbf{0}^{*-1}\mathbf{G}\hat{\mathbf{e}}_\mathbf{0} \tag{19}$$

This may be the only choice of $\mathbf{Z}$ with an economic interpretation.[9] I will return to a deeper analysis of the case of condition (18) where $\mathbf{Z}=\bar{\mathbf{W}}_\mathbf{e}$ in the proof of theorem 2' below.

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[8]Matrix $\bar{\mathbf{W}}_\mathbf{e}$ can be extended to cover cases where e.g. $e^*_{0k}=0$ by assuming that this implies $e^*_k=0$ in general and that column k of $\bar{\mathbf{W}}_\mathbf{e}$ is a null vector. This case could occur in practice for e.g. extractive industries.

[9]From (17) and (8) it is immediately seen that the matrix $\mathbf{Z}$ must satisfy $\mathbf{GZ}=\mathbf{I}$, and of course $\mathbf{e}_0=\mathbf{Z}\mathbf{e}^*_\mathbf{0}$; if, in addition, $\mathbf{Z}$ is required to be nonnegative and of full rank, then necessarily $\mathbf{Z}=\bar{\mathbf{W}}_\mathbf{e}$, see Olsen(2001, note 8).

Using the definitions from (12) and (13) the main result on perfect aggregation is, however

Theorem 2. Perfect aggregation of quantities. The four conditions (20.a)-(20.d) are equivalent

$\mathbf{y}^{*} = \mathbf{G}\mathbf{y}$	for arbitrary $\mathbf{e}$	(20.a)
$\mathbf{x}_{\mathbf{n}}^{*} = \mathbf{G}\mathbf{x}_{\mathbf{n}}$	for arbitrary $\mathbf{e}$	(20.b)
$\mathbf{x}^{*} = \bar{\mathbf{W}}_{\mathbf{a}}\mathbf{x}$	for arbitrary $\mathbf{e}$	(20.c)
$\mathbf{G}\bar{\mathbf{A}}_{\mathbf{c}}\bar{\mathbf{W}}_{\mathbf{x}}'\bar{\mathbf{W}}_{\mathbf{a}} = \mathbf{G}\bar{\mathbf{A}}_{\mathbf{c}}$		(20.d)

Proof: McManus(1956) has shown the equivalence of (20.b) and (20.d).[10] The equivalence of (20.b) and (20.c) follows from (9). It remains to show that they are equivalent to (20.a):

$\mathbf{G}\mathbf{y} = \mathbf{y}^{*}$	for arbitrary $\mathbf{e}$	$\Leftrightarrow$
$\mathbf{G}\bar{\mathbf{y}}(\mathbf{I}-\bar{\mathbf{A}})^{-1} = \bar{\mathbf{y}}^{*}(\mathbf{I}-\bar{\mathbf{A}}^{*})^{-1}\mathbf{G}$		$\Leftrightarrow$
$\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}}(\mathbf{I}-\bar{\mathbf{A}})^{-1} = (\mathbf{I}-\bar{\mathbf{A}}^{*})^{-1}\mathbf{G}$		$\Leftrightarrow$
$(\mathbf{I}-\bar{\mathbf{A}}^{*})\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}} = \mathbf{G}(\mathbf{I}-\bar{\mathbf{A}})$		$\Leftrightarrow$
$(\mathbf{I}-\bar{\mathbf{a}}^{*}-\bar{\mathbf{A}}_{\mathbf{c}}^{*})\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}} = \mathbf{G}(\mathbf{I}-\bar{\mathbf{A}}_{\mathbf{d}}-\bar{\mathbf{A}}_{\mathbf{c}})$		$\Leftrightarrow$
$(\mathbf{I}-\bar{\mathbf{a}}^{*})\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}} = \mathbf{G}(\mathbf{I}-\bar{\mathbf{A}}_{\mathbf{d}}) \quad \wedge \quad \bar{\mathbf{A}}_{\mathbf{c}}^{*}\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}} = \mathbf{G}\bar{\mathbf{A}}_{c}$		$\Leftrightarrow$
$\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}} = \bar{\mathbf{W}}_{\mathbf{a}} \quad \wedge \quad \bar{\mathbf{A}}_{\mathbf{c}}^{*}\bar{\mathbf{W}}_{\mathbf{a}} = \mathbf{G}\bar{\mathbf{A}}_{\mathbf{c}}$		

using (12), (13) and realizing that the equations for intra-branch flows and extra-branch flows are independent of each other, since $\bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}}$ is a (double-weighted) grouping matrix containing zeros in the elements for extra-branch flows . Condition (20.d) follows immediately using (12). The equation to the left is omitted from the theorem since it follows from (20.d) using the identities $\mathbf{i}'\bar{\mathbf{A}}^{*}+\bar{\mathbf{y}}^{*\prime}=\mathbf{i}'$ and $\mathbf{i}'\bar{\mathbf{A}}+\bar{\mathbf{y}}'=\mathbf{i}'$ where $\mathbf{i}$ is the summation vector.[11]□

[10]See McManus(1956, formulae (4b) and (12)).

[11]Define the vector $\mathbf{x}_{\mathbf{c}}=\mathbf{x}-\mathbf{A}_{\mathbf{d}}'\mathbf{i}$, which is a vector of total extra-branch inputs in industries (different from the extra-branch outputs $\mathbf{x}_{\mathbf{n}}=\mathbf{x}-\mathbf{A}_{\mathbf{d}}\mathbf{i}$) and define the adjusted coefficient matrices $\tilde{\mathbf{A}}_{\mathbf{c}}=\mathbf{A}_{\mathbf{c0}}\hat{\mathbf{x}}_{\mathbf{c0}}^{-1}$ and $\tilde{\mathbf{y}}=\hat{\mathbf{y}}_{\mathbf{0}}\hat{\mathbf{x}}_{\mathbf{c0}}^{-1}$. Then $\mathbf{i}'\tilde{\mathbf{A}}_{\mathbf{c}}+\mathbf{i}'\tilde{\mathbf{y}}=\mathbf{i}'$, and (20.d) can be written in the traditional "Hatanaka" form $\tilde{\mathbf{A}}^{*}\mathbf{G} = \mathbf{G}\tilde{\mathbf{A}}_{\mathbf{c}}$ implying that $\mathbf{i}'\tilde{\mathbf{y}}=\mathbf{i}'(\mathbf{I}-\tilde{\mathbf{A}}_{\mathbf{c}})=\mathbf{i}'-\mathbf{i}'\tilde{\mathbf{A}}^{*}\mathbf{G}=\mathbf{i}'\tilde{\mathbf{y}}^{*}\mathbf{G} \Leftrightarrow \mathbf{G}\tilde{\mathbf{y}}=\tilde{\mathbf{y}}^{*}\mathbf{G} \Leftrightarrow \tilde{\mathbf{y}}^{*-1}\mathbf{G}\tilde{\mathbf{y}}=\mathbf{G} \Leftrightarrow \bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}}=\bar{\mathbf{W}}_{\mathbf{a}}$. This deduction also proves the hypothesis in Olsen (2001, note 14) choosing $\tilde{\mathbf{y}}^{*-1}\mathbf{G}\tilde{\mathbf{y}}$ as the adjusted weight matrix.

The interpretation of condition (20.d) is perhaps not immediately clear. But (20.d) can be rewritten, using (12), (10) and (9),

$$\tilde{\mathbf{A}}^*\mathbf{G} = \mathbf{G}\bar{\mathbf{A}}_c(\mathbf{I}-\bar{\mathbf{A}}_d)^{-1}$$

$$= \mathbf{G}\bar{\mathbf{A}}_c(\mathbf{I}-\hat{\mathbf{w}}_d)^{-1}$$

where $\tilde{\mathbf{A}}^*$ is the aggregated net coefficient matrix, as in (10), and $\mathbf{w_d}=\mathbf{i'}\bar{\mathbf{A}}_\mathbf{d}$ (see note 5); this is the familiar form of the "Hatanaka" condition, but since it applies to the extra-branch flows only, the columns of the coefficient matrix $\bar{\mathbf{A}}_\mathbf{c}$ are rescaled using $(\mathbf{I}-\hat{\mathbf{w}}_\mathbf{d})$; the intra-branch flows are unrestricted.

The traditional condition for perfect aggregation, $\mathbf{x}^*=\mathbf{Gx}$, is equivalent to the combination of (20.d) and the additional condition $\bar{\mathbf{a}}^*\mathbf{G}=\mathbf{G}\bar{\mathbf{A}}_\mathbf{d}$, i.e. that all industries grouped into the same main branch must have identical coefficients for aggregated intra-branch flows, see McManus(1956, formula (8)). This unnecessary additional condition would rule out e.g. "vertical" aggregations (along a product line); note that from (13.b) it is equivalent to the condition $\bar{\mathbf{W}}_\mathbf{a}=\mathbf{G}$ confirming that the traditional condition is a special case of (20).

Though Theorem 2 is a useful result there is still a story left to be told, since it states the necessary and sufficient conditions for perfect aggregation for arbitrary fundamental final demand vectors **e**, whereas the basic idea would demand only that they were necessary and sufficient for arbitrary aggregated final demands $\mathbf{e}^*$. Clearly, the conditions of theorems 1 and 2 are still sufficient in this case, but they are not necessary. One reason for this is the possibility of *mixed cases:* If, in the first place, a number of perfect aggregations are possible based on both of the Theorems 1 and 2, then it is possible that some of the aggregates can be further aggregated, because some aggregates based on theorem 1 may fulfil the conditions of theorem 2 and vice versa; such second-level perfect aggregations would not be detected from the conditions (17), (18) or (20) at the first level.[12]

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[12]I have been working on the hypothesis that the mixed cases are the only necessary conditions for arbitrary $\mathbf{e}^*$ not covered by theorems 1 and 2, but I have not been able to prove or disprove so.

4. Perfect aggregation of industries in the price model

The input-output price model is strictly dual to the quantity model. Therefore, the structure of this section is completely the analogous to that of the preceding section, just with all quantity formulae replaced by the dual price formulae. There is one significant difference, though, since in practice final demands by industries can be zero (e.g., for extractive industries), which was not the case for the dual concept of primary inputs by industries; this special property of the final demands introduces a new type of aggregation possibilities, but it also complicates the analysis. In this paper, I will therefore assume that all final demands are positive in the base year, postponing the treatment of zero final demands to a later occasion.

The price model dual to the fundamental quantity model (1), (2) is

$$\mathbf{p}' = \mathbf{p}_y'\bar{\mathbf{y}}(\mathbf{I}-\bar{\mathbf{A}})^{-1} \tag{21}$$

$$\mathbf{p}_e' = \mathbf{p}' \tag{22}$$

where $\mathbf{p}_y$, $\mathbf{p}$ and $\mathbf{p}_e$ are n-vectors of price indexes on primary inputs, production and final demand, respectively. The aggregated price model dual to (3), (4) is

$$\mathbf{p}^{*\prime} = \mathbf{p}_y^{*\prime}\bar{\mathbf{y}}^*(\mathbf{I}-\bar{\mathbf{A}}^*)^{-1} \tag{23}$$

$$\mathbf{p}_e^{*\prime} = \mathbf{p}^{*\prime} \tag{24}$$

where $\mathbf{p}_y^*$, $\mathbf{p}^*$ and $\mathbf{p}_e^*$ are n^*-vectors of corresponding aggregated price indexes, and the aggregated exogenous $\mathbf{p}_y^*$ are defined as fixed-weight aggregates using the base year weights $\bar{\mathbf{W}}_y$, as defined in (16):

$$\mathbf{p}_y^{*\prime} = \mathbf{p}_y'\bar{\mathbf{W}}_y' \tag{25}$$

The problem of aggregation in the price models arises because the aggregated prices $\mathbf{p}^*$ and $\mathbf{p}_e^*$ are generally not equal to the fixed-weight aggregates $\bar{\mathbf{W}}_x\mathbf{p}$ and $\bar{\mathbf{W}}_e\mathbf{p}$, respectively, even though such equalities hold in the base year (where all prices are 1).

The question of gross versus net production is even less significant in the price model than in the quantity model, since the price indexes in the net and gross systems are exactly the same, due to (11). But of course, this fact underpins the point that the weaker conditions restricting only the extra-branch flows must be preferred.

Definition 1'. The aggregation of the price model (21)-(22) into (23)-(24) is (weakly) *perfect* for a given set of vectors of primary input prices $\mathbf{p}_y$ if and only if

$$\mathbf{p}_e^* = \bar{\mathbf{W}}_e\mathbf{p}_e \tag{26}$$

i.e. that the two models yield the same vector of final demand prices by the aggregated industries for that set.

Due to the method of aggregation (5)-(7),(25) the aggregation is perfect by definition for $\mathbf{p}_y = \mathbf{p}_{y0} = \mathbf{i}$ (and for all vectors $\mathbf{p}_y$ proportional to $\mathbf{i}$), but in general it is not, and pursuing the basic idea I will seek the conditions for perfect aggregation for arbitrary (nonnegative) aggregated vectors $\mathbf{p}_y^* = \bar{\mathbf{W}}_y\mathbf{p}_y$.

Though this concept of perfect aggregation is the proper operationalization of the basic idea, it is, like in the primal case, possible to define a stronger concept requiring that the endogenous prices from the *fundamental* model can be determined from the aggregated model:

Definition 2'. The aggregation of the price model (21)-(22) into (23)-(24) is *superperfect* for a given set of vectors of primary input prices if and only if

$$\mathbf{p}_e = \mathbf{G}'\mathbf{p}_e^* \tag{27}$$

i.e. that the two models yield the same vector of prices on final demand by the fundamental industries for that set. It is immediately clear that, as in the primal case,

- superperfect aggregation of prices is a special case of perfect aggregation of prices, since (27) implies (26) but not vice versa.
- the aggregation (23)-(25) is superperfect for all vectors $\mathbf{p}_y$ proportional to $\mathbf{p}_y$ in the base year (which is the unit vector $\mathbf{i}$)
- no nontrivial aggregation can be superperfect for arbitrary $\mathbf{p}_y$, due to the smaller dimensionality of the aggregated variables.

Following the basic idea I will seek the conditions for (weakly) perfect aggregation for arbitrary aggregated primary input price vectors $\mathbf{p}_y^*$, but results for other domains and for superperefect aggregation are developed in the process.

Lemma 1'. An aggregation of the price model is superperfect if and only if

$$\mathbf{p} = \mathbf{G}'\mathbf{p}^*$$

i.e. the production prices are proportional in the groups of $\mathbf{G}$.

Proof: Follows immediately from (27) using (22) and (24).

Theorem 1'. An aggregation of the price model (21),(22) into (23),(24) is superperfect for arbitrary $\mathbf{p}_y^*$ if and only if there exists a matrix **Q** such that the conditions (28) and (29) are both satisfied:

$$\mathbf{p}_y' = \mathbf{p}_y^{*\prime}\mathbf{Q} \qquad \text{arbitrary } \mathbf{p}_y^* \qquad (28)$$

$$(\mathbf{I}-\bar{\mathbf{A}}^*)\bar{\mathbf{y}}^{*-1}\mathbf{Q}\bar{\mathbf{y}} = \mathbf{G}(\mathbf{I}-\bar{\mathbf{A}}) \qquad (29)$$

Proof:
Is completely dual to the proof of theorem 1 and therefore omitted.
□

A valid choice for **Q** would be the matrix **G**, in which case (29) is equivalent to the condition (20.d) of Theorem 2. This may be the only economically meaningful choice of **Q**.[13]

Theorem 2'. Perfect aggregation of prices. The three conditions (30.a)-(30.c) are equivalent

$$\mathbf{p}_e^{*\prime} = \mathbf{p}_e'\bar{\mathbf{W}}_e' \qquad \text{arbitrary } \mathbf{p}_y \qquad (30.a)$$

$$\mathbf{p}^{*\prime} = \mathbf{p}'\tilde{\mathbf{W}}_x' \qquad \text{arbitrary } \mathbf{p}_y \qquad (30.b)$$

$$\tilde{\mathbf{W}}_x'\mathbf{G}\bar{\mathbf{A}}_c\bar{\mathbf{W}}_x' = \bar{\mathbf{A}}_c\bar{\mathbf{W}}_x' \qquad (30.c)$$

Proof:[14]

$$\mathbf{p}_e'\bar{\mathbf{W}}_e' = \mathbf{p}_e^{*\prime} \qquad \text{arbitrary } \mathbf{p}_y' \qquad \Leftrightarrow$$

$$\mathbf{p}'\bar{\mathbf{W}}_e' = \mathbf{p}^{*\prime} \qquad \text{arbitrary } \mathbf{p}_y' \qquad \Leftrightarrow$$

$$\bar{\mathbf{y}}(\mathbf{I}-\bar{\mathbf{A}})^{-1}\bar{\mathbf{W}}_e' = \bar{\mathbf{y}}\bar{\mathbf{W}}_x'(\mathbf{I}-\bar{\mathbf{A}}^*)^{-1} \qquad \Leftrightarrow$$

$$(\mathbf{I}-\bar{\mathbf{A}})^{-1}\bar{\mathbf{W}}_e' = \bar{\mathbf{W}}_x'(\mathbf{I}-\bar{\mathbf{A}}^*)^{-1} \qquad \Leftrightarrow$$

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[13]From (25) and (28) it is immediately seen that the matrix **Q** must satisfy $\mathbf{Q}\bar{\mathbf{W}}_y'=\mathbf{I}$ and, since price indexes are 1 in the base year, that the column sums of **Q** must be 1. If, in addition, **Q** is required to be nonnegative (which is equivalent to a requirement of nonnegative prices) and of full rank, then necessarily **Q**=**G**, see Olsen(2001, note 12).

[14]The proof is completely dual to the proof of theorem 2.

$$\mathbf{\bar{W}'_e(I-\bar{A}^*) = (I-\bar{A})\bar{W}'_x} \qquad \Leftrightarrow$$
$$\mathbf{\bar{W}'_e(I-\bar{a}^*-G\bar{A}_c\bar{W}'_x) = (I-\bar{A}_d-\bar{A}_c)\bar{W}'_x} \qquad \Leftrightarrow$$
$$\mathbf{\bar{W}'_e(I-\bar{a}^*) = (I-\bar{A}_d)\bar{W}'_x} \ \wedge \ \mathbf{\bar{W}'_e G\bar{A}_c\bar{W}'_x = \bar{A}_c\bar{W}'_x} \qquad \Leftrightarrow$$
$$\mathbf{\bar{W}'_e = (I-\bar{A}_d)\bar{W}'_x(I-\bar{a}^*)^{-1} = \tilde{W}'_x} \ \wedge \ \mathbf{\tilde{W}'_x G\bar{A}_c\bar{W}'_x = \bar{A}_c\bar{W}'_x}$$

which proves the equivalence of (30.a) and (30.c) immediately, and (30.b) substituting (24) and $\mathbf{\tilde{W}'_x=\bar{W}'_e}$ into (30.a). The equation to the left is omitted from the theorem since it follows from (30.c) using the identities $\mathbf{A^*_c i+e^*=x^*_n}$ and $\mathbf{A_c i+e=x_n}$ where **i** is the summation vector.[15] □

The interpretation of condition (30.c) is perhaps not immediately clear; however, it can be written, using (6) and (12)

$$\mathbf{\tilde{W}'_x A^*_{c0} = A_{c0}G'}$$

where $\mathbf{A^*_{c0}=GA_{c0}G'}$ (=$\mathbf{A^*_0-\hat{a}^*_0}$) is the aggregated net flow matrix in the base year. Thus, the semi-aggregated matrix of extra-branch flows, $\mathbf{A_{c0}G'}$, must have proportional rows within the groups of **G**. This is clearly a dual form of the "Hatanaka" conditions, but like in the primal case the restriction applies to the extra-branch flows only; the intra-branch flows are unrestricted.[16]

As in the primal case of the quantity model, there is still a story left to be told, since theorem 2' states the necessary and sufficient conditions for perfect aggregation for arbitrary fundamental primary input prices $\mathbf{p_y}$, whereas the basic idea would demand only that they were necessary and sufficient for arbitrary aggregated primary input prices $\mathbf{p^*_y}$. Clearly, the conditions of theorems 1' and 2' are still sufficient in this case, but they are not necessary. As in the case of the quantity models, a reason for this is the possibility of *mixed cases.*[17]

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[15]Since (30.c) can be written $\mathbf{\tilde{W}'_x\bar{A}^*_c = \bar{A}_c\bar{W}'_x}$ using (12), and $\mathbf{\bar{A}^*_c x^*=x^*_n-e^*}$, they imply that $\mathbf{\tilde{W}'_x\bar{A}^*_c x^*=\tilde{W}'_x(x^*_n-e^*)} \Leftrightarrow \mathbf{\bar{A}_c\bar{W}'_x x^*_0=\tilde{W}'_x(x^*_{n0}-e^*_0)} \Leftrightarrow \mathbf{A_{c0}i=x_{n0}-\tilde{W}'_x e^*_0} \Leftrightarrow \mathbf{x_{n0}-e_0=x_{n0}-\tilde{W}'_x e^*_0} \Leftrightarrow \mathbf{e_0=\tilde{W}'_x e^*_0} \Leftrightarrow \mathbf{\tilde{W}'_x=W'_e}$. This also proves the hypothesis in Olsen(2001, note 11).

[16]The more conventional price aggregation formula $\mathbf{p^{*\prime}=p'\bar{W}'_x}$ applies only in the special case where, in addition to (30.c), $\mathbf{\tilde{W}'_x\hat{a}^*_0=\bar{A}_{d0}G'_x}$ holds; this in turn would imply that $\mathbf{\bar{W}'_x=\tilde{W}'_x}$, from (14). This reflects that if only (30.c) hold, then the price of the diagonal flow is not identical to the output price.

[17]I have been working on the hypothesis that the mixed cases are the only necessary conditions not covered by theorems 1' and 2', but I have not been able to prove or disprove so.

5. General perfect aggregation of input-output models and bias measures

The results concerning quantity and price models are now combined to yield the general conditions for aggregation of a dual pair of input-output models.

Definition 3. The aggregation of a dual pair of input-output quantity and price models (1),(2),(21),(22) into the aggregated models (3),(4),(23),(24) is *perfect* for a given set of vectors of final demands **e** and primary input prices $\mathbf{p_y}$ if and only if the two models yield the same vector of primary inputs and final demand prices by the aggregated industries for that set, i.e. that

$$\mathbf{y}^* = \mathbf{G}\mathbf{y}$$

$$\mathbf{p}_e^* = \bar{\mathbf{W}}_e\mathbf{p}$$

Theorem 3. A sufficient condition for perfect aggregation of a dual pair of input-output price and quantity models is ((31)∧(20))∨((30)∧(32)), where

$$\mathbf{p}_y' = \mathbf{p}_y^{*\prime}\mathbf{G} \qquad \text{arbitrary } \mathbf{p}_y' \tag{31}$$

$$\mathbf{e} = \tilde{\mathbf{W}}_x'\mathbf{e}^* \qquad \text{arbitrary } \mathbf{e}^* \tag{32}$$

Proof: Follows from the fact that (30) fulfils (18) and (32) fulfils (17) implying that the aggregation of quantities is superperfect and the aggregation of prices is perfect; likewise, (20) fulfils (29) and (31) fulfils (28) implying that the aggregation of prices is superperfect and the aggregation of quantities is perfect.

Again there is a possibility of mixed cases.

This result completes the structural interpretation of the result of Olsen(1993) that in general perfect aggregration requires either proportional quantities or identical prices (or both).[18]

The relaxation of the traditional conditions into the weaker conditions of theorem 3 is not just a theoretical exercise. While the traditional conditions allow only "horizontal" aggregation (of different product chains), the relaxation of the conditions of intra-branch supplies opens the possibilities of "vertical" aggregation (along a single product chain), which are quite relevant in practice. However, even after this relaxation few empirical input-output models will satisfy the conditions exactly. Instead, the case of perfect aggregation is used as a baseline point for

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[18]Note that the conditions of theorem 3 imply that $\bar{\mathbf{W}}_a = \bar{\mathbf{y}}^{*-1}\mathbf{G}\bar{\mathbf{y}}$ and $\tilde{\mathbf{W}}_x' = \bar{\mathbf{W}}_e'$, as shown in the proofs of theorems 2 and 2’, respectively.

measuring the aggregation bias, which is traditionally defined as $\beta_x = \mathbf{x}^* - \mathbf{G}\mathbf{x}$ and should be evaluated for relevant values of final demand **e**. However, an immediate consequence of theorems 1 and 2 is that this measure should properly be redefined as[19]

$$\beta_x = \mathbf{x}^* - \bar{\mathbf{W}}_a \mathbf{x} \qquad (33)$$

enabling many more possibilities of good aggregations, supposedly of the "vertical" type. Alternatively, the bias could be measured on the primary inputs instead, as

$$\beta_y = \mathbf{y}^* - \mathbf{G}\mathbf{y} \qquad (34)$$

which will yield similar results, due to theorem 2.

A complete evaluation of an aggregation should also include the price biases defined as either $\beta_p = \mathbf{p}^* - \tilde{\mathbf{W}}_x \mathbf{p}$ or $\beta_e = \mathbf{p}^* - \bar{\mathbf{W}}_e \mathbf{p}$.

A commonly applied measure of aggregation error was defined by Theil(1957) as the "first order bias" $\beta_1 = \bar{\mathbf{A}}^* \mathbf{G} - \mathbf{G}\bar{\mathbf{A}}$. This measure should not be used since if the domain of **e** is restricted it is not related to the general quality of an aggregation in any simple manner, see Olsen(2001); this is unfortunate, since in practice the domain of **e** is restricted by consumer preferences etc. At least the first order measure should be redefined as $\beta_1 = \bar{\mathbf{A}}_c^* \bar{\mathbf{W}}_a - \mathbf{G}\bar{\mathbf{A}}_c$, from (20.d).

6. Conclusions

The standard conditions for perfect aggregation of input-output models are too narrow, since they require unnecessary restrictions on the intra-branch flows. Instead, the general conditions were derived, leading to a revised definition of aggregation bias meaures. The use of the commonly applied "first order aggregation bias" measure is not recommended.

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[19]If very a large number of possible aggregations is analyzed it could be a problem that the computation of matrix $\bar{\mathbf{W}}_a$ is slightly more complicated than the usual aggregator **G**. This could be addressed by using the approximation $\beta_{x2} = \mathbf{x}^* - \bar{\mathbf{y}}^{*-1} \mathbf{G} \bar{\mathbf{y}} \mathbf{x}$, since in case of perfect aggregation the two measures are identical and zero.

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